Supplementary material

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# A. Operational Constraints of the Aggregator

An aggregator can integrate DERs, including distributed generation , demand response , and distributed energy storage . They are limited by (A.1), (A.2), and (A.3)-(A.5), respectively.

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where , are the lower and upper limits of distributed generation's active power, respectively;, are the lower and upper limits of demand response's active power, respectively; is the energy state of aggregator 's distributed energy storage at bus ; , are the lower and upper power limits of the distributed energy storage, respectively; , are the lower and upper energy limits, respectively; , are the charging and discharging efficiency, respectively; is the discretization step.

# B. Proof of Proposition 2.

The models of the aggregator and utility in stochastic Stackelberg game, i.e., the upper and lower level optimization problems of (30), can be expressed below.

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| *s.t.*  (29b), (29e) |  |
| *s.t.*  (29f) |  |

Denote, as the optimal solution of (B.1) and , as the optimal solution of (B.2). Denote the set of data passed by the aggregator to utility by . Denote the set of data passed by the utility to aggregator by . The stochastic Stackelberg game equilibrium is reached when the following equation is satisfied:

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In other words, if we can find a set of solutions satisfying the above two equations, the game equilibrium is reached.

Based on the variational inequality [2], if , is the optimal solution of (B.1), then

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Similarly, if , is the optimal solution of (B.2), then

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That is, (B.3)-(B.6) constitute a sufficient and necessary condition of the stochastic Stackelberg game equilibrium.

Let , be the optimal solution of (31). For , , the following formula should be satisfied:

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whose specific detail is as follows:

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If , , , are the solutions to the game equilibrium, should equal in (B.5). Similarly, the leasing price is equal to the sum of and , and is equal to . In (B.6), the leasing capacities , equal and , respectively. This ensures that the formulas (B.5)(B.6) always hold.

If we let

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whose specific detail is as follows:

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Then, formulas (B.5)(B.6) can become (B.8). That is, stochastic Stackelberg game equilibrium is the optimal solution of (30), where the SES leasing prices and capacities in and equal to dual variables.

Similarly, if , is the optimal solution of (31), let

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Then, the optimality conditions (B.5)(B.6) hold. (31) satisfies the game equilibrium of (B.5)(B.6), which can be solved by ADMM.

References

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2. D. Yan and Y. Chen, "Distributed coordination of charging stations with shared energy storage in a distribution network," *IEEE Trans. Smart Grid*, vol. 14, no. 6, pp. 4666-4682, Nov. 2023.