Supplementary material

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# A. Operational Constraints of the Aggregator

An aggregator can integrate DERs to generate an aggregated virtual synchronous generator (VSG) and virtual battery (VB), whose model can be shown as follows：

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Demands and DER generations are aggregated into a VSG, which can be formulated by (A.2) (e.g., through the approach in [1]). Battery energy storage systems are aggregated into a VB, whose constraints are shown in (A.3)-(A.5).

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where , are the limits of VSG's active power, respectively; , are the active power output and energy state of aggregator 's VB, respectively; , are the lower and upper power limits, respectively; , are the lower and upper energy limits, respectively; , are the charging and discharging efficiency, respectively; is the discretization step. Denote this set of constraints by .

# B. Proof of Proposition 2.

The models of the aggregator and utility in stochastic Stackelberg game, i.e., the upper and lower level optimization problems of (36), can be expressed below.

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| *s.t.*  (36b), (36e) |  |
| *s.t.*  (36f) |  |

Denote, as the optimal solution of (B.1) and , as the optimal solution of (B.2). Denote the set of data passed by the aggregator to utility by . Denote the set of data passed by the utility to aggregator by . The stochastic Stackelberg game equilibrium is reached when the following equation is satisfied:

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In other words, if we can find a set of solutions satisfying the above two equations, the game equilibrium is reached.

Based on the variational inequality [2], if , is the optimal solution of (B.1), then

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Similarly, if , is the optimal solution of (B.2), then

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That is, (B.3)-(B.6) constitute a sufficient and necessary condition of the stochastic Stackelberg game equilibrium.

Let , be the optimal solution of (37). For , , the following formula should be satisfied:

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whose specific detail is as follows:

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If , , , are the solutions to the game equilibrium, should equal in (B.5). Similarly, the leasing price is equal to the sum of and , and is equal to . In (B.6), the leasing capacities , equal and , respectively. This ensures that the formulas (B.5)(B.6) always hold.

If we let

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whose specific detail is as follows:

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Then, formulas (B.5)(B.6) can become (B.8). That is, stochastic Stackelberg game equilibrium is the optimal solution of (36), where the SES leasing prices and capacities in and equal to dual variables.

Similarly, if , is the optimal solution of (37), let

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Then, the optimality conditions (B.5)(B.6) hold. (37) satisfy the game equilibrium of (B.5)(B.6), which can be solved by ADMM.

References

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2. D. Yan and Y. Chen, "Distributed coordination of charging stations with shared energy storage in a distribution network," *IEEE Trans. Smart Grid*, vol. 14, no. 6, pp. 4666-4682, Nov. 2023.